MATH 1730 - PROBLEMS FROM 3.1 TO 3.5

1) $y = e^{-x^2+8x}$

$g(f(x))$

$e^{(\ )}$

$\frac{dy}{dx} = e^{-x^2+8x} \cdot (-2x+8)$

$= -2x\left(e^{(\ )}\right) + 8e^{(\ )}$

2) $y = e^{\sqrt{x-7}} \longleftrightarrow (x-7)^{1/2}$

$\frac{dy}{dx} = e^{\sqrt{x-7}} \cdot \frac{1}{2} \frac{1}{\sqrt{x-7}}$

3) $y = (5x^2 - 8x)\, e^{x^2-4x}$

$u \qquad v$

$u = 5x^2 - 8x \qquad u' = 10x - 8$

$v = e^{x^2-4x} \qquad v' = e^{x^2-4x}(2x-4)$

$\frac{dy}{dx} = u'v + uv' = (10x-8)e^{x^2-4x} + (2x+4)(5x^2-8x)e^{x^2-4x}$

$\cdot e^{x^2-4x}$

$$\frac{d}{dx} a^x = \ln(a) \cdot a^x$$

4) $y = x^5 (3)^x$ | $u'v + uv'$

u v

$$\frac{dy}{dx} = 5x^4 \cdot 3^x + x^5 \cdot \ln 3 \cdot 3^x$$ | at least 2 decimal places

5) $y = 7^{x^4+2}$ | $\ln 7 (x^4+2)$

$$\frac{dy}{dx} = (x^4+2)\ln 7 \{4x^3\}$$

$$= 4x^3 (x^4+2) \ln 7$$

6) $y = x^4 \ln x - \frac{1}{2}x^2$

$\frac{dy}{dx} = -x + 4x^3 \ln x + \frac{x^4}{x}$

$= 4x^3 \ln x + x^3 - x$

$\ln(ab) = \ln a + \ln b$

7) $y = \ln(6x)$

$= \ln 6 + \ln x$

(i) $\frac{dy}{dx} = 0 + \frac{1}{x}$

(ii) $\frac{dy}{dx} = \frac{1}{6x} \cdot 6 = \frac{1}{x}$

$\frac{u'v - uv'}{v^2}$

8) $y = \frac{\ln x}{x^4}$

$u = \ln x \qquad u' = \frac{1}{x}$

$v = x^4 \qquad v' = 4x^3$

$\frac{dy}{dx} = \frac{x^3 - 4x^3 \cdot \ln x}{x^8} = \frac{1 - 4\ln x}{x^5}$

9) $y = \ln(7x^2 + 5x + 2)$

$$\frac{dy}{dx} = \frac{1}{7x^2 + 5x + 2}(14x + 5)$$

10) $y = (\ln x)^4 \neq \ln x^4$

$$\frac{dy}{dx} = 4(\ln x)^3 \cdot \frac{1}{x}$$

11) $y = (x^2 - x)\ln 6x$

$x = 2$

$y = ?$

$m = ?$

equation of the line tangent to the graph at x=2

$$\frac{dy}{dx} = (2x - 1)\cdot \ln 6x + (x^2 - x)\cdot \frac{1}{x}$$

$f(2) = y$ at $x = 2 \Rightarrow 4.9698$

m at $x = 2 = (2 \times 2 - 1)\ln 12 + (2^2 - 2)\cdot \frac{1}{2}$

$\simeq 8.45$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 4.97 = 8.45(x - 2)$

$$\frac{d}{dx}\log_a x = \frac{1}{\ln a}\cdot\frac{1}{x}$$

12) $y = \log_6(5x+1)$

$$\frac{dy}{dx} = \frac{1}{\ln 6}\cdot\frac{1}{5x+1}\cdot 5$$

13) $y = \log_8(x^3+x)$

$$\frac{dy}{dx} = \frac{1}{\ln 8}\cdot\frac{1}{x^3+x}\left(3x^2+1\right)$$

14) $y = 4\log_7(\sqrt{x}-2)$ $\qquad x^{1/2}$

$$f' = \frac{1}{2}x^{-1/2}$$

$$\frac{dy}{dx} = \frac{4}{\ln 7}\cdot\frac{1}{\sqrt{x}-2}\cdot\frac{1}{2\sqrt{x}} = \frac{2}{\sqrt{x}\left(\sqrt{x}-2\right)\ln 7}$$

15) $y = 5^x \cdot \log_2 x$

$u \quad v$

$u = 5^x \qquad u' = \ln 5 \cdot 5^x$

$v = \log_2 x \qquad v' = \frac{1}{\ln 2} \cdot \frac{1}{x}$

$\boxed{u'v + uv'}$

$\frac{dy}{dx} = 5^x \ln 5 \cdot \log_2 x + \frac{5^x}{x \ln 2}$

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franchise expansion - rate of increase in the number of shops in a franchise is 10%

1) eqn: $\frac{dN}{dt} = 0.10N$ | $t=0, N=50$

2) find the function that satisfies this equation:

$$N(t) = 50e^{0.10t}$$

$P_0 \; P(0) \; c$

3) how many franchises will there be in 20 years?

$$N(20) = 50 \cdot e^{0.10 \times 20} = 50e^{2}$$

4) in what period will it double?

$$T = \frac{\ln 2}{K} = \frac{\ln 2}{0.10} =$$

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iodine 131 has a decay rate of 9.6% per day, starting with 500

1) $\frac{dQ}{dt} = -0.096Q$

2) $Q(t) = 500e^{-0.096t}$

3) at $t = 4$ days, $Q(4) = 500e^{-0.096 \times 4}$

4) time taken to halve the quantity of I-131

$$T = \frac{\ln 2}{K} = \ln 2 / 0.096 =$$

(18) Euler bank advertises that it compunds interest continuously and that it will double your money in 15 years. what is the rate of interest? (annual rate).

$$T = \frac{\ln 2}{k}$$

$T \to 15$, $k \to ?$

$$\Rightarrow k = \frac{\ln 2}{T} = \frac{\ln 2}{15} = 4.62\%$$

(19) growth rate of the demand for coal in the workd is 4% per year. when will the demand be double that of 2006?

$$T = \frac{\ln 2}{4} = 17.33$$

$$2006 + 17.33 = 2023$$